DEVELOPMENT OF REGIONAL TAPER AND VOLUME EQUATIONS: HARDWOOD SPECIES

Authors: Aaron Weiskittel and Rongxia Li

Introduction

Last year, taper equations for the primary softwood species in the region were presented (Li *et al.*, 2011). The primary goal of this analysis was to compare and evaluate two taper equations for the major hardwood species in the region. One equations was from the work of Li et al. (2011), while the other was recently developed for the majority of hardwood species in the Northeastern United States (Westfall and Scott, 2010). We aimed to find out whether the two well-performed taper equations can be directly applied to the hardwood species in the Acadian Region. In addition, the taper equations were compared to the widely used Honer (1965) regional volume equations and bark thickness equations are presented for species with sufficient data.

Methods

Data

Stem analysis data were gathered mainly from four sources: (1) 683 trees used in the Honer (1965) study, (2) 38 trees collected by Georgia Pacific in New Brunswick, (3) 2429 trees collected by the Quebec Ministry of Natural Resources; and (4) 795 trees used by Westfall and Scott (2010) (table 3-12). The majority of the data was obtained from the stem analysis of felled trees, while the Westfall and Scott (2010) was collected on standing trees. Preliminary analysis indicated relatively little influence of the different data sources. Sample plots were primarily located in eastern and central Maine, New Brunswick, and Quebec. The Westfall and Scott (2010) data did also include data from other northeastern states like New York, Ohio, Pennsylvania, and West Virginia. Again, preliminary analysis indicated relatively little influence of geographic region on the results.

On each sampled tree, measurements of diameter outside bark (dob) were taken at stump height, breast height, and approximately every 1 m or 2 m after breast height until the top of the main stem (where branches spited the main stem). In a few of the datasets, measurements of diameter inside bark were also obtained.

Analysis

Give then results of Li et al. (2011) on softwood species, the Kozak (2004) model form was selected for use in this analysis (equation 3-3). The other taper equation was originally formulated by Valentine and Gregoire (2001) and then revised by Westfall and Scott (2010).

We used nonlinear mixed-effects modeling (nlme) techniques to estimate parameters in equation 3-5 (table 3-13). The parameters α_0 and β_3 were chosen to include random effects since the models with these two as mixed parameters gave the minimum AIC among all other models with two random-effects. The parameter estimation procedure was implemented by the nlme function in the nlme library in R (Pinheiro and Bates, 2009). The estimated parameter values for the Westfall and Scott (2010) equation are found in table 3-14. For each sample tree, Smalian's formula was used to calculate both estimated volume outside bark (VOB) based on estimated dob and observed volume based on measured dob.

Since volume inside bark (VIB) is usually of greater interest, bark thickness equations for the different hardwood species were developed when sufficient data was available. Although Li and Weiskittel (2011) found that bark thickness equations dependent on the ratio of inside bark DBH to outside bark DBH were superior to other equations, bark thickness is generally not measured and using an equation was more effective than assuming a fixed ratio between dob and diameter inside bark (dib).

Consequently, the following equation was estimated

dib= $_{1}$dob ^{2}

where the $ß_i$'s are parameters to be estimated and all other variables have been defined above.

For evaluation, mean absolute bias (MAB), root mean square error (RMSE), and mean bias (MB) were used to evaluate model performance for both diameter and volume prediction.

Results

The number of sample trees ranged from 6 (green ash) to 1327 (yellow birch) (table 3-12). The taper equation using the Kozak (2004) model form fit well as the RMSE ranged from 0.88 cm (gray birch) to 3.45 cm (yellow birch) (tables 3-13 and 3-14). The Kozak (2004) equation performed slightly better than the Westfall and Scott (2010) equation. Overall, the Kozak (2004) equation resulted in 71.3, 7.4, and 12.2% reduction in MB, MAB, and RMSE, respectively. The improvements were most noticeable in balsam poplar, green ash, and sugar maple.

Overall, the three approaches for predicting total stem volume were quite similar (table 3-15). Generally, the Kozak (2004) taper equations show the lowest bias, while the Westfall and Scott (2010) had the highest bias. For some species, the Honer (1965) volume equation outperformed the taper equations, particularly red oak and yellow birch. Volume prediction differences were most notably in trees larger than 25 cm (10 inches) (figure 3-18).

The bark thickness equations fit well with RMSEs between 0.28 cm (sugar maple) to 0.69 cm (yellow birch) (table 3-16). Across the different species tested, quaking aspen was predicted to have the thickest bark for a given dob, while sugar maple was predicted to have the thinnest.

Equation 3-5:

$$d = \alpha_0 D^{\alpha_1} H^{\alpha_2} X^{\beta_1 z^4 + \beta_2 (1/e^{D/H}) + \beta_3 X^{0.1} + \beta_4 (1/D) + \beta_5 H^Q + \beta_6 X}$$

where $X = \frac{1-(h/H)^{1/3}}{1-p^{1/3}}$, $Q = 1 - z^{1/3}$, p=1.3/H, *h* is the stem section height, *d* is the diameter outside bark at height *h*, H and D refers total tree height and dob at breast height, and $\alpha_0 - \alpha_2$, $\beta_1 - \beta_6$ represent parameters to be estimated.

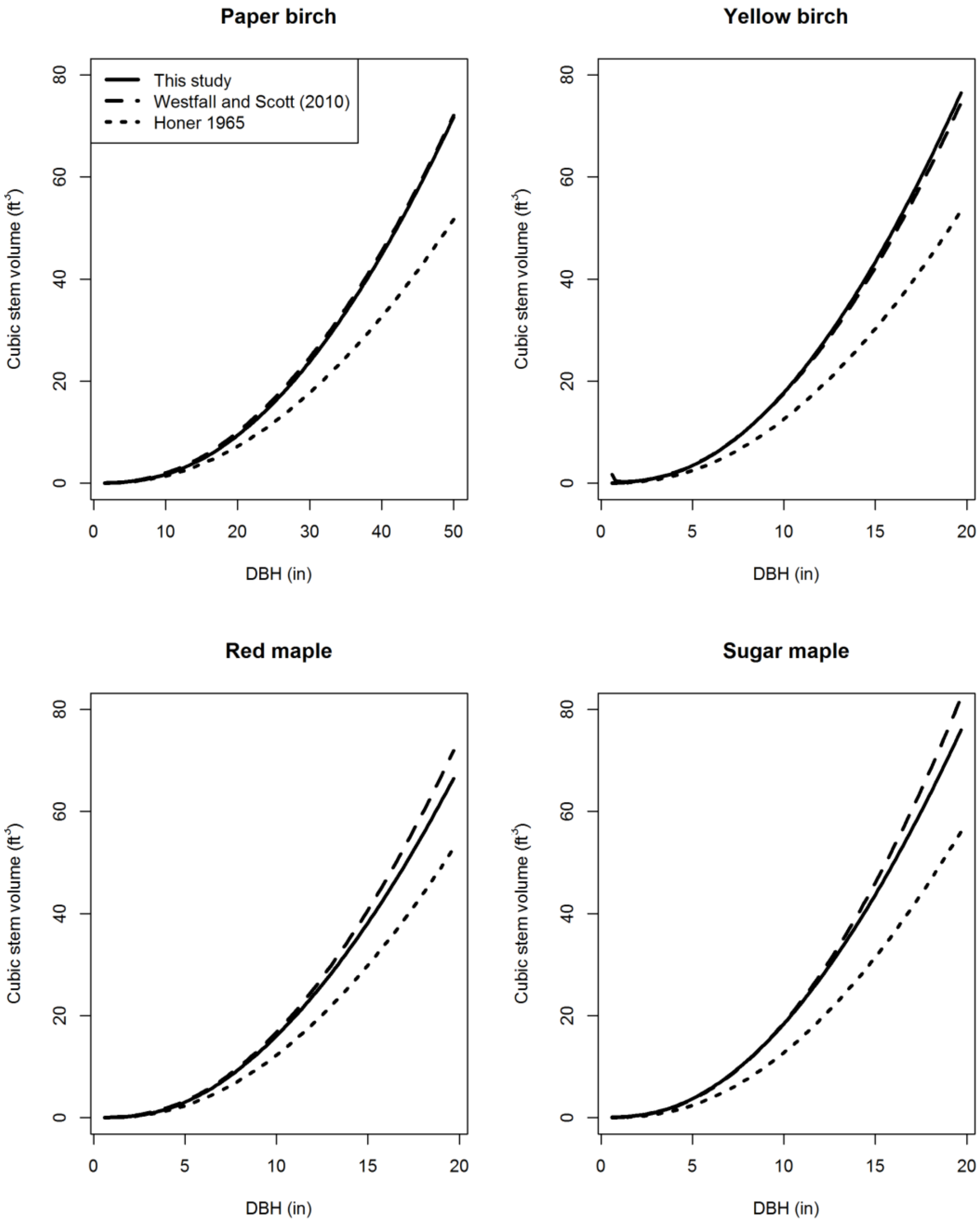

Figure 3-18. Estimated total stem volume (ft^3) for paper birch, yellow birch, red maple, and sugar maple using the taper equations in this study as well as Westfall and Scott (2010) and the volume equations of Honer (1965) across a range of DBH classes (in).

***Table 3-12. Attributes (mean ± SD; minimum and maximum in parentheses) of the sample trees by species*.**

Species	# of Sample Trees	# of dob (dib) measurements	DBH (cm)	HT (m)	dob (cm)	dib (cm)
American beech	142	2285	32.4 ± 17.4 (7.1, 89.7)	18.93 ± 6.12 (7.89, 33.77)	22.8 ± 18.2 (0.1, 107.7)	-
Black cherry	115	1808	28.6 ± 14.2 (7.6, 81.8)	19.66 ± 5.51 (5.85, 32.49)	19.8 ± 15.0 (0.1, 95.3)	-
Balsam poplar	9	123	23.3 ± 8.4 (9.6, 34.79)	16.42 ± 3.97 (9.72, 21.61)	16.5 ± 10.1 (0.1, 38.6)	-
Bigtooth aspen	26	382	26.0 ± 9.8 (9.1, 42.7)	20.49 ± 5.35 (10.82, 31.58)	18.1 ± 11.7 (0.1, 52.6)	-
Green ash	6	76	20.3 ± 11.7 (8.9, 38.6)	16.23 ± 7.19 (10.24, 26.42)	15.5 ± 12.3 (0.1, 54.6)	-
Gray birch	13	141	15.2 ± 5.3 (8.1, 24.4)	12.27 ± 2.83 (7.58, 16.86)	11.4 ± 7.1 (0.1, 29.5)	
Paper birch	991	12961 (3271)	26.9 ± 9.63 (5.6, 70.8)	16.57 ± 3.22 (6.5, 26.8)	25.1 ± 10.9 (0.1, 86.7)	15.1 ± 7.5 (0.7, 50.3)
Quaking aspen	41	643 (161)	24.8 ± 9.5 (8.4, 39.1)	18.28 ± 4.34 (7.80, 24.57)	18.1 ± 11.1 (0.1, 49.5)	21.1 ± 9.6 (2.3, 46.7)
Red maple	162	2580 (145)	32.3 ± 13.5 (7.6, 75.9)	19.18 ± 4.83 (6.83, 32.00)	24.9 ± 15.2 (0.1, 104.4)	19.1 ± 9.7 (1.7, 46.1)
Red oak	46	768	35.1 ± 19.8 (13.2, 112.5)	21.62 ± 5.55 (12.25, 32.83)	24.6 ± 21.3 (0.1. 164.3)	-
Sweet birch	41	587	24.3 ± 14.3 (8.1, 69.3)	17.15 ± 4.76 (8.96, 29.02)	17.9 ± 15.1 (0.1, 100.6)	-
Sugar maple	975	15999 (105)	37.4 ± 11.2 (7.8, 79.7)	20.32 ± 3.46 (7.68, 31.91)	33.1 ± 13.1 (0.1, 94.7)	19.1 ± 11.3 (2.4, 50.1)
White ash	46	787 (159)	28.4 ± 15.6 (9.1, 76.4)	21.02 ± 5.93 (8.22, 38.19)	20.7 ± 16.3 (0.1, 105.9)	18.4 ± 10.6 (2.0, 59.4)
Yellow birch	1327	17077 (2913)	38.3 ± 14.5 (8.1, 89.5)	17.64 ± 3.27 (8.32, 27.14)	36.5 ± 16.3 (0.1, 170.4)	18.9 ± 10.4 (2.7, 84.6)

Table 3-13. Parameter estimates for the Kozak (2004) taper equation (equation 3-3).

Species	Parameters								
	α_0	α_1	α_2	β_1	β_2	β_3	β_4	β_5	β_6
American beech	1.0683	0.9975	-0.0128	0.3921	-1.0546	0.7702	4.1035	0.1186	-1.0807
Black cherry	0.9824	0.9901	0.0215	0.6093	-0.5463	0.5054	1.6561	0.0409	-0.3028
Balsam poplar	1.0036	0.7442	0.2876	0.6634	-2.0048	0.7508	3.9248	0.0277	-0.1309
Bigtooth aspen	1.0194	1.0055	-0.0110	0.5105	-1.3264	0.5132	7.2108	0.0711	-0.5718
Green ash	1.0852	1.1862	-0.2262	0.5199	1.4303	-0.3495	3.1953	0.1392	-0.2967
Gray birch	1.0050	0.8836	0.1308	0.6114	-0.1142	0.2521	2.6574	0.0590	-0.1751
Paper birch	0.7161	0.9811	0.1383	0.4782	0.3092	0.2643	-0.3021	0.0859	-0.2787
Quaking aspen	0.5527	0.9048	0.3075	0.7131	-0.5883	0.3620	2.8516	0.0382	-0.1343
Red maple	0.7458	1.0092	0.0891	0.5862	-0.8659	0.6414	3.0604	0.0828	-0.6486
Red oak	1.1729	1.0225	-0.0699	0.4506	-0.9029	0.5927	3.6267	0.1656	-1.1143
Sweet birch	0.8471	0.9875	0.0770	0.9323	-0.9546	0.4855	3.0295	0.0768	-0.2384
Sugar maple	1.0456	0.9613	0.0386	0.8556	-0.2497	0.3889	1.2548	0.0413	-0.1135
White ash	0.8551	0.9769	0.0770	0.7819	-0.7918	0.4767	3.5004	0.0859	-0.4880
Yellow birch	1.1017	0.9485	0.0371	0.7663	-0.0281	0.1788	4.8570	0.0753	-0.2051

Table 3-14. Mean bias (MB), mean absolute bias (MAB), and root mean square error (RMSE) for the Kozak (2004) and Westfall and Scott (2010) taper equations in predicting diameter outside bark (cm). Only the fixed effects for the Kozak (2004) equations were used.

Species	Kozak (2004)			Westfall and Scott (2010)		
	MB (cm)	MAB (cm)	RMSE (cm)	MB (cm)	MAB (cm)	RMSE (cm)
American beech	0.1476	2.0120	2.9453	-0.5514	2.0336	3.1795
Black cherry	-0.0172	1.5429	2.3098	-0.0753	1.5842	2.4577
Balsam poplar	-0.1206	0.9962	1.2720	-0.2708	1.1877	1.7751
Bigtooth aspen	-0.0809	1.2983	1.9448	0.4205	1.5271	2.2863
Green ash	0.0256	0.9683	1.3206	0.8538	1.6006	2.4422
Gray birch	-0.0522	0.6705	0.8814	0.0845	0.6745	0.9778
Paper birch	-0.0435	1.6228	2.3043	0.0326	1.7342	2.5168
Quaking aspen	0.0442	1.6019	2.1694	0.2411	1.5823	2.2554
Red maple	0.0309	1.9850	2.9151	-0.2007	2.0428	3.0321
Red oak	0.1899	2.0712	3.2869	-0.0837	2.0033	3.2795
Sweet birch	0.04257	1.4699	2.5264	-0.2710	1.4543	2.7108
Sugar maple	-0.1772	2.5088	3.3429	-1.3549	3.0848	4.5731
White ash	0.1516	1.7189	2.8149	-0.2766	1.7506	3.0723
Yellow birch	-0.7912	2.3933	3.4463	-0.8194	2.4159	3.5717
Overall	-0.0465	1.6329	2.3914	-0.1622	1.7626	2.7236

Table 3-15. Mean bias (MB), mean absolute bias (MAB), and root mean square error (RMSE) for the Kozak (2004)and Westfall and Scott (2010) taper equations as well as the Honer (1965) volume equations in predicting total tree volume. Only the fixed effects for the Kozak (2004) equations were used.

Species	Kozak (2004)			Westfall and Scott (2010)			Honer (1965)		
	MB (m^3)	MAB (m^3)	RMSE (m^3)	MB (m^3)	MAB (m^3)	RMSE (m^3)	MB (m^3)	MAB (m^3)	RMSE (m^3)
American beech	0.0265	0.0979	0.2016	-0.0457	0.0986	0.2105	-0.1221	0.1424	0.2890
Black cherry	0.0088	0.0681	0.1237	0.0005	0.0773	0.1456	-0.0614	0.1034	0.2173
Balsam poplar	-0.0048	0.0319	0.0463	-0.0248	0.0512	0.0875	-0.0189	0.0532	0.0861
Bigtooth aspen	-0.0008	0.05103	0.0815	0.0562	0.0859	0.1396	0.0564	0.0841	0.1334
Green ash	0.0063	0.0123	0.0146	0.0252	0.04458	0.0731	-	-	-
Gray birch	0.0007	0.0078	0.0120	0.0012	0.0079	0.0121	-0.0001	0.0076	0.0012
Paper birch	-0.1079	0.1275	0.2317	-0.0833	0.1091	0.1982	-0.0575	0.0973	0.1727
Quaking aspen	-0.0066	0.0516	0.0764	0.0144	0.0568	0.0793	0.0127	0.0538	0.0761
Red maple	-0.0337	0.1013	0.1639	-0.0436	0.1115	0.1732	-0.0239	0.1061	0.1712
Red oak	0.0432	0.1375	0.3611	-0.0123	0.1239	0.2423	0.0692	0.1441	0.2983
Sweet birch	0.0017	0.0567	0.1487	-0.0281	0.0583	0.1763	0.0039	0.0523	0.1206
Sugar maple	-0.1670	0.2033	0.3122	-0.2367	0.2557	0.3122	-0.1064	0.1767	0.2768
White ash	0.0052	0.0796	0.1483	-0.0267	0.0824	0.1471	-	-	-
Yellow birch	-0.3623	0.3705	0.5637	-0.3309	0.3407	0.5237	-0.2769	0.2920	0.4588
Overall	-0.0422	0.0998	0.1776	-0.0525	0.1074	0.1801	-0.0438	0.1094	0.1918

Table 3-16. Parameter estimates, mean bias and root mean square error (RMSE) for the bark thickness equation by species.

Species	$ß_1$	$ß_2$	Mean bias (cm)	RMSE (cm)
Paper birch	0.8969	1.0179	-0.0591	0.3467
Quaking aspen	0.8449	1.0332	-0.1103	0.6679
Red maple	0.9214	1.0117	-0.0161	0.5501
Sugar maple	0.9383	1.0064	-0.0026	0.2787
White ash	0.8834	1.0188	0.0036	0.4669
Yellow birch	0.8688	1.0275	-0.2223	0.6873

Discussion

Hardwood species represent a more significant challenge for modeling stem form and volume when compared to softwoods. This is because a significant portion of their total volume can be in branches rather than a main bole. For example, MacFarlane (2010) found that branches comprised between 5 to 21% of the total tree volume for common hardwood species in the Lake States. Consequently, the taper equations presented in this analysis will likely underestimate total tree volume as measurements of bole volume were only available for analysis. Unfortunately, the Westfall and Scott (2010) equation suffers from a similar shortcoming. Future efforts will be needed to better quantify the ratio between branch and bole volume.

Both the Kozak (2004) and Westfall and Scott (2010) taper equations predicted hardwood stem form quite accurately, despite the range of species and tree sizes in the data. The Kozak (2004) did predict dob slightly better, on average, than the Westfall and Scott (2010). However, this improved accuracy wasn't realized when used to estimate observed stem volume as the equations had a similar level of bias. Both taper equations did generally perform slightly better than the Honer (1965) equations, except for certain species like red oak and yellow birch. However, the taper equations will likely prove more effective for estimating merchantable stem volume.

Inside bark observations were limited for most species and nonexistent for others. This suggests the need to collect bark thickness information for additional hardwood species in the future. Additional data from the New Brunswick Department of Natural Resources is currently being entered and will be used to update the presented equations. In addition, the New Brunswick Growth and Yield Unit is planning to stem section several hardwood trees in the upcoming summer, which will also be used to update the equations. Regardless, additional measurements in Maine would be helpful

.